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Digital Twin – Mechanical Energy

1 Scope

This document describes the different algorithm used in the digital Twin

2 Analysis type : GPS Replay

2.1 Energy calculation

It is assumed that the GPS tracks $\boldsymbol{x}(t)$ are describing the position of the Center of Gravity (CoG) of the vehicle in function of time, in the Earth Coordinate System (ECS).

The vehicle COG velocity in the ECS is:

$$\boldsymbol{v}(t) = \dot{\boldsymbol{x}}(t)$$

The orientation of the vehicle in the ECS:

- $\boldsymbol{e_1}$ is collinear to the velocity

$$\boldsymbol{e_1}(t) = \begin{cases} if\ \|\boldsymbol{v}(t)\| \neq 0, & \dfrac{\boldsymbol{v}(t)}{\|\boldsymbol{v}(t)\|} = \begin{Bmatrix} e_{1x} \\ e_{1y} \\ e_{1z} \end{Bmatrix} \\ else\ if\ \|\boldsymbol{v}(t)\| = 0\ and\ t > t_0, & \boldsymbol{e_1}(t_{i-1}) \\ else, & \begin{Bmatrix} 1 \\ 0 \\ 0 \end{Bmatrix} \end{cases}$$

- $\boldsymbol{e_2}$ is oriented to the left of the vehicle and parallel to the $x_E y_E$ plan

$$\boldsymbol{e_2}(t) = \frac{1}{\sqrt{e_{1y}^2 + e_{1x}^2}} \begin{Bmatrix} -e_{1y} \\ e_{1x} \\ 0 \end{Bmatrix}$$

- $\boldsymbol{e_3}$ is defined perpendicular to the two other vectors

$$\boldsymbol{e_3}(t) = \boldsymbol{e_1} \times \boldsymbol{e_2}$$

The vehicle angular velocity in the Vehicle Coordinate System (VCE) as:

$$\boldsymbol{\omega}_{VCS}(t) = (\dot{\boldsymbol{e}}_{\boldsymbol{2}}^T \cdot \boldsymbol{e_3}) \cdot \boldsymbol{e_1} + (\dot{\boldsymbol{e}}_{\boldsymbol{3}}^T \cdot \boldsymbol{e_1}) \cdot \boldsymbol{e_2} + (\dot{\boldsymbol{e}}_{\boldsymbol{1}}^T \cdot \boldsymbol{e_2}) \cdot \boldsymbol{e_3}$$

The total vehicle kinetic energy in the ECS is given by:

$$E_k(t) = \frac{1}{2} m \|\boldsymbol{v}_{ECS}(t)\|^2 + \frac{1}{2} \boldsymbol{\omega}_{VCS}(t)^T \cdot \boldsymbol{I}_{VCS} \cdot \boldsymbol{\omega}_{VCS}(t)$$

With:

m: the vehicle mass

$I_{VCS} = \begin{bmatrix} I_{11} & I_{12} & I_{13} \\ I_{12} & I_{22} & I_{23} \\ I_{13} & I_{23} & I_{33} \end{bmatrix}$: the 3x3 inertia matrix in the VCS

Applying the theorem of kinetic energy to the vehicle at the time t_1, and neglecting the contribution of the internal forces, gives

$$\Delta E_k(t_i) = \int_{t_{i-1}}^{t_i} \boldsymbol{F_{ext}}(t) \cdot \boldsymbol{v}(t) \cdot dt$$

Where:

$$\Delta E_k(t_i) = E_k(t_i) - E_k(t_{i-1})$$

$$\boldsymbol{F_{ext}}(t) = \boldsymbol{F_{aero}}(t) + \boldsymbol{F_{roll}}(t) + \boldsymbol{F_{ground}}(t) + \boldsymbol{F_{gravity}}(t) + \boldsymbol{F_{mec.motor}}(t)$$

Note that the above forces must be in the ECS.

The work produced by the motor $W_{motor.mec}$ between time t_{i-1} and t_i is given by

$$\boxed{W_{motor.mec}(t_i) = \Delta E_k(t_i) - \int_{t_{i-1}}^{t_i} (\boldsymbol{F_{aero}}(t) + \boldsymbol{F_{gravity}}(t) + \boldsymbol{F_{ground}}(t) + \boldsymbol{F_{rolling}}(t)) \cdot \boldsymbol{v}(t) \cdot dt}$$

The expression of each component of the external forces is given in the following sections:

2.1.1 Aerodynamic Force $\boldsymbol{F_{aero}}$

The aerodynamic force in the ECS is given by:

$$\boldsymbol{F_{aero}}(t) = -\frac{1}{2}\rho v(t)^2 \cdot Cx \cdot S \cdot \boldsymbol{e_1}(t)$$

2.1.2 Gravity Force $\boldsymbol{F_{gravity}}$

The gravity force in the ECS is given by:

$$\boldsymbol{F_{gravity}}(t) = m \cdot g \cdot \begin{Bmatrix} 0 \\ 0 \\ -1 \end{Bmatrix}$$

2.1.3 Ground reaction force $\boldsymbol{F_{ground}}$

To compute the ground reaction force, the angle between the velocity vector and the horizontal plan shall be calculated. The *road plane elevation angle* λ is calculated as:

$$\lambda = arcsin\left(\frac{e_{1z}}{\sqrt{e_{1x}^2 + e_{1y}^2 + e_{1z}^2}}\right)$$

NB: the road plane elevation angle has a sign. It is positive when driving uphill, and negative when driving downhill.

The reaction of the ground is given in the ECS as:

$$F_{ground}(t) = m \cdot g \cdot \cos(\lambda) \cdot \boldsymbol{e_3}$$

2.1.4 Rolling Force $\boldsymbol{F_{rolling}}$

The rolling force in the ECS is proportional to the total ground force, but opposite the velocity direction:

$$F_{rolling}(t) = -C_{rr} \cdot\ m \cdot g \cdot \cos(\lambda) \cdot \boldsymbol{e_1}$$

2.2 Power Calculation

The motor mechanical power is calculated from the motor mechanical work as follow:

$$P_{motor.mec} = \frac{d}{dt} W_{motor.mec}(t)$$

2.3 Discretisation

The vehicle CoG position is given at discrete times:

$$\boldsymbol{x}(t) = \boldsymbol{x}(t_0), \boldsymbol{x}(t_1), \dots, \boldsymbol{x}(t_i), \dots \boldsymbol{x}(t_n)$$

The derivative and integral of the different variable (position, force, energy) can thus be computed assuming different interpolation scheme. The following methods are implemented

2.3.1 Linear

Derivatives are computed as follows:

$$\frac{d}{dt} x(t_i) = \frac{x(t_i) - x(t_{i-1})}{t_i - t_{i-1}}$$

The integrations are computed as follows:

$$\int_{t_{i-1}}^{t_i} x(t)dt = \frac{1}{2}(x_i + x_{i-i})(t_i - t_{i-1})$$

